ERTK
EXTC

Sem III
C.B.G.S.
AM-III

27/11/15

QP Code : 5106

(Revised course)

Time :3 hours — Total marks :80

N.B : (1) Question No.1 is compulsory.
(2) Answer any three questions from remaining.
(3) Assume suitable data if necessary.

1. (a) Evaluate

$$\int_0^\infty e^{-t}\left(\frac{\cos 3t-\cos 2t}{t}\right)dt$$ 05

(b) Obtain the Fourier Series expression for $f(x)=2x-1$ in (0,3) 05

(c) Find the value of 'p' such that the function $f(z)=\frac{1}{2}\log(x^2+y^2)+i\tan^{-1}\left(\frac{py}{x}\right)$ is analytic. 05

(d) If $\bar{F}=(y\sin z-\sin x)\hat{i}+(x\sin z+2yz)\hat{j}+(xy\cos z+y^2)\hat{k}$.
Show that $\bar{F}$ is irrotational .Also find its scalar potential. 05

2. (a) Solve the differential equation using Laplace Transform 06

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=3te^{-t}$$, given y(0)=4 and y'(0)=2

(b) Prove that 06

$$J_4(x)=\left(\frac{48}{x^3}-\frac{8}{x}\right)J_1(x)-\left(\frac{24}{x^2}-1\right)J_0(x)$$

(c) i) In what direction is the directional derivative of $\phi=x^2y^2z^4$ at (3,-1,-2) maximum. Find its magnitude. 08
ii) If $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$
Prove that $\nabla r^n=nr^{n-2}\bar{r}$

[TURN OVER

3. (a) Obtain the Fourier Series expansion for the function 06

$$f(x) = 1 + \frac{2x}{\pi}, -\pi \le x \le 0$$

$$= 1 - \frac{2x}{\pi}, 0 \le x \le \pi$$

(b) Find an analytic function f(z) =u+iv where. 06

$$u - v = \frac{x-y}{x^2+4xy+y^2}$$

(c) Find Laplace transform of 08

i) $\cosh t \int_0^t e^u \sinh u$

ii) $t\sqrt{1+\sin t}$

4. (a) Obtain the complex form of Fourier series for 06
$f(x) = e^{ax}$ in (-L,L)

(b) Prove that 06
$\int x^4 J_1(x)dx = x^4 J_1(x) - 2x^3 J_3(x) + c$

(c) Find 08

i) $L^{-1}\left[\frac{2s-1}{s^2+4s+29}\right]$

ii) $L^{-1}\left[\cot^{-1}\left(\frac{s+3}{2}\right)\right]$

5. (a) Find the Bi- linear Transformation which maps the points 1,i,-1 of z plane onto 0,1,∞ of w-plane 06

(b) Using Convolution theorem find 06

$$L^{-1}\left[\frac{s^2}{\left(s^2+4\right)^2}\right]$$

[TURN OVER

(c) Verify Green's Theorem for $\int_C \overline{F}.\overline{dr}$ where $\overline{F} = (x^2 - y^2)\hat{i} + (x+y)\hat{j}$ and C is the triangle with vertices (0,0) ,(1,1) and (2,1) 08

6. (a) Obtain half range sine series for 06

$$f(x) = x, 0 \le x \le 2$$
$$= 4 - x, 2 \le x \le 4$$

(b) Prove that the transformation $w = \frac{1}{z+i}$ transforms the real axis of the z-plane into a circle in the w-plane. 06

(c) i) Use Stoke's Theorem to evaluate $\int_C \overline{F}.\overline{dr}$ where $\overline{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ and C is the rectangle in the plane z=0,bounded by x=0,y=0,x=a and y=b. 08

ii) Use Gauss Divergence Theorem to evaluate $\iint_S \overline{F}.\hat{n}ds$ where $\overline{F} = 4x\hat{i} + 3y\hat{j} - 2z\hat{k}$ and S is the surface bounded by x=0,y=0,z=0 and 2x+2y+z=4
